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Nuclear Instruments and Methods in Physics Research A 482 (2002) 565–566

**NUCLEAR
INSTRUMENTS
& METHODS
IN PHYSICS
RESEARCH**
Section A

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Letter to the Editor

A plausible mathematical interpretation of the ‘variance’ spectra obtained with the DSPEC^{PLUS}™ digital spectrometer

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Received 31 October 2001; accepted 10 January 2002

Abstract

This paper presents a simple mathematical explanation why the variance spectrum implemented in the DSPEC^{PLUS}™ digital spectrometer represents a good approximation of the statistical counting uncertainty of the loss-corrected spectra obtained with the so-called ‘zero dead time’ technique. © 2002 Elsevier Science B.V. All rights reserved.

PACS: 02.50.Cw; 02.90.+p; 07.85.-m*Keywords:* Nuclear; Uncertainty; Zero dead time; Loss-free counting; Variance

1. Introduction

The DSPEC^{PLUS}™ is a digital spectrometer with a built-in pulse loss correction method, called “zero dead time” (ZDT™) counting, which involves the regular addition of virtual counts to the true spectrum (add- n operation instead of add-1) according to a short-term ratio of real time to ‘system live time’ ($n = RT/LT$) [1]. This method is comparable to the well-known “loss-free counting” (LFC) method [2] made available by competing constructors, a difference probably being the duration of the lifetime period during which an updated value of the weighing factor n is determined. Both techniques restore the linearity of the spectrometer up to high count rates, indifferent to the type of count loss, the average count rate and the variation of the count rate with

time. They are of particular interest to applications in which live time correction techniques are inadequate, like with high-rate counting procedures involving nuclides with half-lives that are short compared to the counting time (e.g. in the field of neutron activation analysis) or when the radioactivity is physically moving (e.g. real time monitoring of effluents).

An important novelty implemented in the DSPEC^{PLUS}™ is the so-called ‘variance’ spectrum taken in parallel with the ZDT spectrum. It consists of the squares of the ‘ n ’-values registered in the corresponding channels of the ZDT-spectrum. The constructor claims that this Σn^2 -spectrum represents the variance associated with the number of counts, Σn , in the ZDT-corrected spectrum [1]. The mathematical arguments in support of this claim have not yet been published, as a consequence of tactical decisions involved with the patenting of the method.

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In this paper, an independent and simple mathematical interpretation of the ‘variance spectrum’ is presented.

2. The ‘variance spectrum’

In recent work [3], it has been shown that the statistical uncertainty associated with ‘zero dead time’ spectrometry is similar to that of ‘loss-free counting’. Hence it can be approximated by the following mathematical formula [4]:

$$\frac{\sigma(N_{\text{ZDT}})}{\sqrt{\langle n \rangle N_{\text{ZDT}}}} = r \sqrt{1 + \left(\frac{\sigma(n)}{\langle n \rangle} \right)^2} \quad (1)$$

in which $N_{\text{ZDT}} = \Sigma n$ is the number of counts in the considered ‘region of interest’ (ROI) of the ZDT spectrum, r is a correction factor (often close to 1) due to leading edge pileup and $\langle n \rangle$ is the ‘average’ loss-correction factor in the ROI. To avoid confusion, the interpretation of the latter needs to be further specified.

The averaging of the loss-correction factor $\langle n \rangle$ takes place in time, between the start and stop of the measurement. It is a ‘weighted’ average in the sense that it is taken over every count (true or virtual) in the considered ROI of the ZDT spectrum. It is NOT calculated from the ratio of the ZDT counts to the true counts in an uncorrected spectrum, as would be done in the ‘dual spectrum’ method [2]. The latter ratio corresponds to an averaging of n over every count in the uncorrected spectrum, which is an underestimate of the $\langle n \rangle$ meant in Eq. (1).

The mathematical expression for $\langle n \rangle$ is readily found to be

$$\langle n \rangle = \frac{\sum nn}{\sum n} = \frac{\sum n^2}{N_{\text{ZDT}}} \quad (2)$$

Eq. (2) instantly provides the meaning to be attached to the Σn^2 -spectrum, since it can be rewritten in the form

$$\sum n^2 = \langle n \rangle N_{\text{ZDT}} \quad (3)$$

It turns out that the ‘sum of squares’ spectrum corresponds to a first approximation of the statistical variance of N_{ZDT} as represented by

Eq. (1), i.e. $\sigma^2(N_{\text{ZDT}}) \approx \langle n \rangle N_{\text{ZDT}}$. Hence, the name ‘variance spectrum’ is more or less justified, setting aside the omission of two correction terms.

There is experimental evidence supporting this mathematical interpretation of the Σn^2 -spectrum. It has been demonstrated that the variance spectrum predicts the counting uncertainty well in small parts of γ -ray spectra taken at high (variable or fixed) count rates [3]. In such small ROIs, the correction factor r equals 1. As a consequence, one can conclude that the standard deviation $\sigma(n)$ appearing in the second correction factor of Eq. (1) is relatively low. This points to a rather long refreshing period of several milliseconds rather than microseconds for a new correction factor n to be adopted. For larger ROIs, corresponding to a significant fraction of the total spectrum, the Σn^2 -spectrum gives an underestimate of the true statistical variation. This can exactly be accounted for by the correction factor r , which corroborates the statement following from Eq. (3) that the Σn^2 -spectrum yields a first order approximation of the true variance.

3. Conclusions

The relationship between the ‘sum of squares’ or Σn^2 -spectrum and the statistical uncertainty in the corresponding ‘zero dead time’ spectrum of the DSPEC^{PLUS}™ digital spectrometer has been established in a simple mathematical way. There is a nearly direct link with the true statistical variance, setting aside the omission of two minor correction factors. Therefore, the Σn^2 -spectrum is quite successful in predicting the statistical uncertainty involved with ZDT spectrometry.

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