

Choosing the Right TAC

The following topics provide the information needed for selecting the right time-to-amplitude converter (TAC) for the task. The basic principles of operation are described, and the critical operating characteristics are delineated. The selection guide chart provides a quick reference to the major features of the full range of ORTEC models.

Timing with TACs

When a timing application demands picosecond precision, a time-to-amplitude converter is a prime candidate. A TAC can achieve such exceptional precision because it uses an analog technique to convert small time intervals to pulse amplitudes. Figure 1 illustrates the principle. (Although the actual circuitry in a TAC employs sophisticated transistor switches, the devices in Fig. 1 have been represented as toggle switches for a simpler description.)

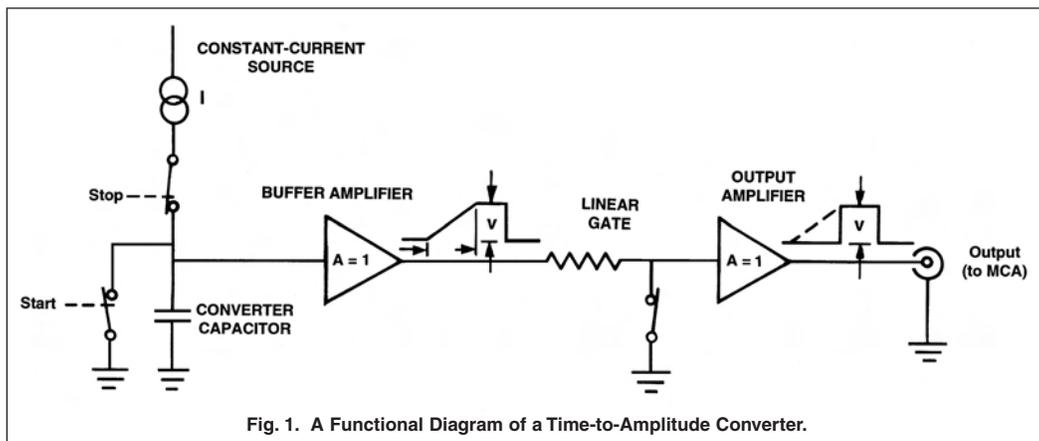


Fig. 1. A Functional Diagram of a Time-to-Amplitude Converter.

Before a time measurement starts, all the switches in Fig. 1 are closed. The arrival of the leading edge of the "start" signal opens the "start" switch, and the converter capacitor begins to charge at a rate set by the constant-current source. The leading edge of the "stop" signal opens the "stop" switch and prevents any further charging of the capacitor. Because the charging current I is constant, the voltage developed on the capacitor is given by

$$V = \frac{I t}{C} \quad (1)$$

where t is the time interval between start and stop pulses and C is the capacitance of the converter capacitor. Consequently, the voltage is proportional to the time interval. This voltage pulse is passed through the buffer amplifier to the linear gate. A short time after the stop pulse arrives, the linear gate switch opens to pass the voltage pulse through the output amplifier to the TAC output. After a few microseconds, all the switches return to the closed condition. This terminates the output pulse and discharges the capacitor to ground potential in preparation for the next pair of start and stop events. The result is a rectangular output pulse with a width of a few microseconds and an amplitude that is proportional to the time interval between the start and stop pulses. This pulse is typically fed to an ADC or a multichannel analyzer for pulse-height measurement.

As the conversion and measurement process is repeated for additional pairs of start and stop pulses, a time spectrum grows in the multichannel analyzer memory. The shape of this spectrum will depend on the time correlations between the start and stop events. For strongly correlated events, as experienced in gamma-gamma coincidence experiments, the spectrum is usually a well-defined peak with a shape that is nearly Gaussian. In fluorescence lifetime measurements, the time peak has a sharp rise at "zero" time followed by an exponential decay. In the case of totally uncorrelated start and stop events, the shape of the spectrum is determined by the Interval Distribution, which describes the probability of the length of time intervals between randomly arriving events.¹ If n_{start} is the number of valid start pulses accepted by the TAC and MCA during the measurement of the time spectrum, and r_{stop} is the average counting rate of the random, uncorrelated stop pulses, the number of counts recorded between times t and $t + dt$ in the time spectrum will be

$$dN = n_{\text{start}} r_{\text{stop}} e^{(-r_{\text{stop}} t)} dt \quad (2)$$

If r_{stop} is very small compared to the reciprocal of the TAC time range, the spectrum from the uncorrelated events will appear to be a flat background.

Typically, the start and stop inputs of time-to-amplitude converters are designed to accept the fast logic signals from timing discriminators. Each timing discriminator, in turn, derives its signal from the amplified output of some type of detector or transducer. On the shortest time ranges, time-to-amplitude converters can deliver exceptionally fine time resolution (~10 ps). Under such circumstances, the controlling factors for time resolution are normally the timing jitter and walk contributed by the sources of the start and stop signals.

¹Ron Jenkins, R. W. Gould, Dale Gedcke, Quantitative X-Ray Spectrometry, Marcel Dekker, New York, 1981, First Edition, Chapter 4.

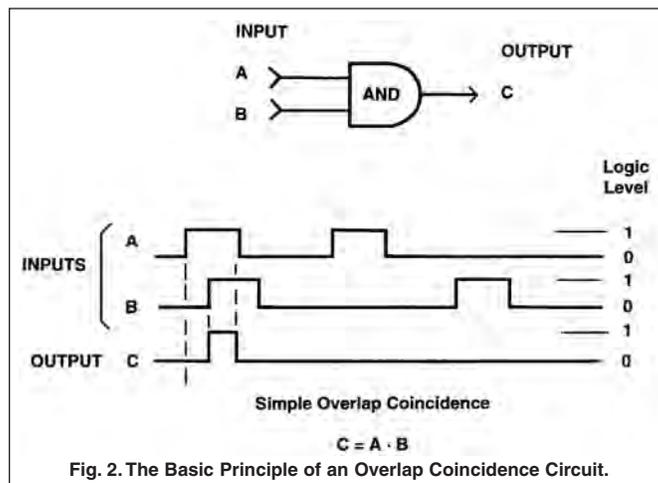
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Adding Delays and Biased Amplifiers

Because of the nature of the TAC circuitry, it is difficult to measure time intervals <10 ns with good linearity. However, many measurements involve start and stop signals that arrive within ± 10 ns of each other. The solution for these situations is to insert an appropriate delay in the stop signal path. Selecting a delay in the range of 10 to 30 ns on an ORTEC Model 425A Nanosecond Delay is usually sufficient to move the timing peak into the linear region of the time spectrum. The stop delay can also be adjusted to center the features of interest in the time spectrum.

A TAC Makes Coincidence Set-Ups Much Easier

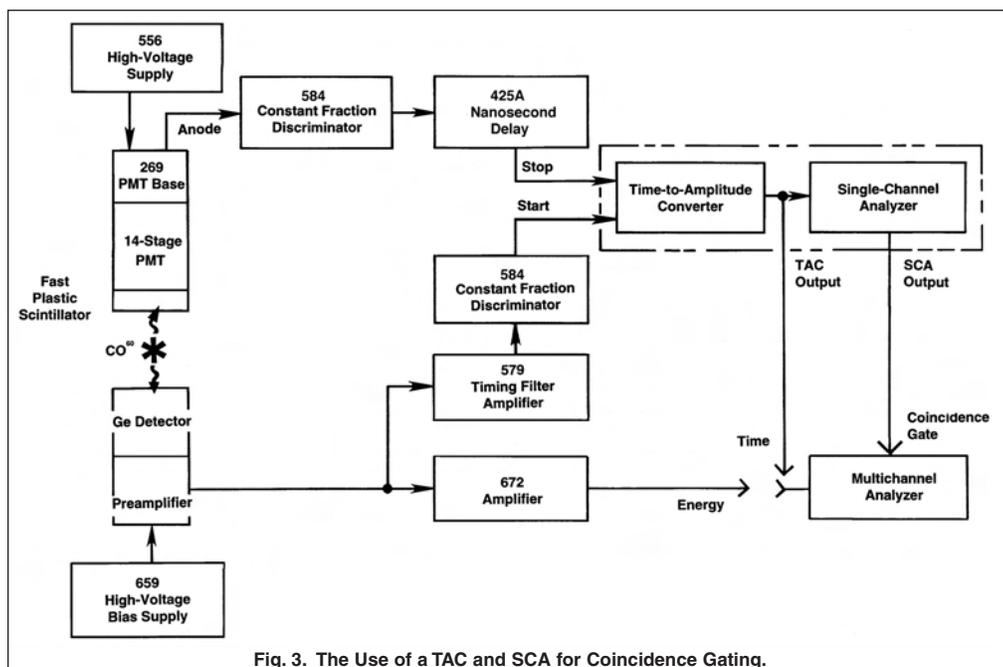
By adding a single-channel pulse-height analyzer (SCA) to the output of a TAC, the time-to-amplitude converter can be used to identify coincident events between two detectors. To appreciate the power of this method, one must compare it to the alternative technique, the simple overlap coincidence circuit. Figure 2 illustrates the principle behind the overlap coincidence function offered in the ORTEC Models CO4020, 414A, and 418A. The overlap coincidence circuit is simply a two-input AND gate. As depicted by the waveforms in Fig. 2, the AND gate generates a "logic 1" output only when "logic 1" pulses are present on both the A and B inputs. In fact, the output is generated only for the time during which the A and B pulses overlap. This is the reason the circuit is known as an overlap coincidence.



Detecting truly coincident pulses places special restrictions on pulses A and B. First, the delays through the electronics producing the pulses must be the same for both detectors, so that both pulses arrive at the AND gate at the same time. Second, the width of each pulse must be equal to the maximum timing uncertainty for its respective detector. If the pulse width is too narrow or the delays are not quite matched, some of the truly correlated pulses will not overlap, and the C output will be missing. This represents a loss of coincidence detection efficiency. If the A or B pulses are too wide, uncorrelated events will have a higher probability of generating an output due to accidental overlap, and that is contrary to the purpose of the scheme. Choosing the proper pulse widths and delays to achieve 100% efficiency for identifying correlated events, while minimizing the sensitivity to uncorrelated events, requires a laborious series of trial-and-error measurements. Experimenters often avoid this task by making the pulse widths much larger than the "best guess" for the detector's timing uncertainty. Of course, the quality of the experiment will suffer if these pulses are either too wide or too narrow.

Figure 3 shows how a TAC with an SCA (i.e., Model 567 TAC/SCA) can be used to simplify the selection of the optimum coincidence resolving time. The prompt timing pulse from the germanium detector operates the start input of the TAC, while the delayed pulse from the scintillation detector triggers the stop input. When the analog output of the TAC is analyzed by the multichannel analyzer, the spectrum in Fig. 4 is observed. There is a peak formed by the correlated gamma-ray events from the two detectors. This peak sits on an essentially flat background caused by the uncorrelated events from the two detectors. (See the comments following Equation 2.)

By connecting the logic output of the SCA to the gate input of the MCA,



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only those TAC pulses which fall within the SCA window will be analyzed by the MCA. With minimal effort, the SCA thresholds can be adjusted to ensure that only the events in the peak are accepted. Subsequently, the SCA output is used as the coincidence gate when analyzing the energy spectrum from the germanium detector on the MCA. By replacing the overlap coincidence with a TAC and SCA, the optimum coincidence resolving time can be selected quickly and with full knowledge of the intrinsic time resolution of the system.

Note that the SCA window for "correlated events" in Fig. 4 includes a background contribution from "uncorrelated events". The contribution of these uncorrelated events to the energy spectrum can be assessed by setting another SCA window of equal width in the uncorrelated background region of the time spectrum. This second SCA is used to gate a second MCA, which will record the energy spectrum corresponding to uncorrelated events. Subtraction of the two energy spectra will yield a spectrum free of the uncorrelated events. (NOTE: A minor correction to the second SCA window width based on Equation 2 may be required at high counting rates.)

At extremely high counting rates the processing time of the TAC and SCA may contribute noticeably to the dead time losses of the coincidence spectrometer. In this rare case, an overlap coincidence with updating inputs and outputs is the better choice because of its inherently lower dead time for identifying coincident events.

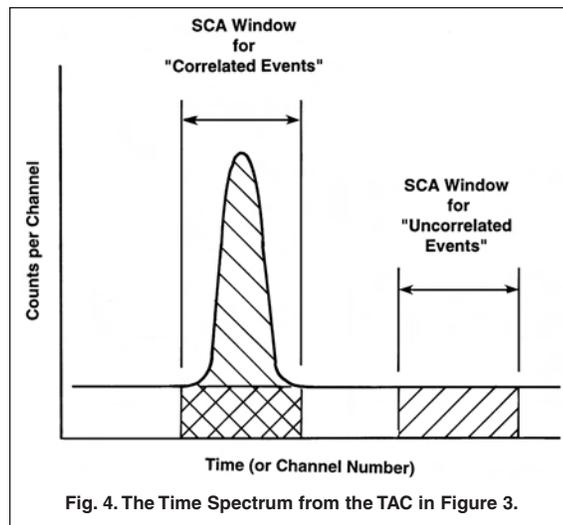


Fig. 4. The Time Spectrum from the TAC in Figure 3.

Assigning Start and Stop Inputs for Lower Dead Time

If a very high counting rate is provided to the start input while an extremely low counting rate is supplied to the stop input, the TAC will spend a lot of time responding to start pulses that have no associated stop pulse within the selected time range. Starts with no stops will cause excessive dead time in the TAC without producing useful data. Reversing the input assignments so that the higher counting rate is on the stop input will minimize this dead time.

Reversing the start and stop inputs is particularly important in applications where a sample is excited by a periodic pulse and the time spectrum of the reaction products emitted by the sample is to be recorded. Usually, the repetition rate of the periodic pulse is high and the counting rate of the reaction products is extremely low. Logically, one would expect the excitation pulse to be the start pulse and the reaction products to provide the stop pulses. But, this creates too much dead time in the TAC. To reduce the dead time, the reaction products should drive the start input while the excitation pulse is delayed and fed to the stop input. The length of the stop delay should be approximately 90% of the time range selected on the TAC. Fig. 5 is an example of the reversed start/stop technique applied to a fluorescence lifetime spectrometer.

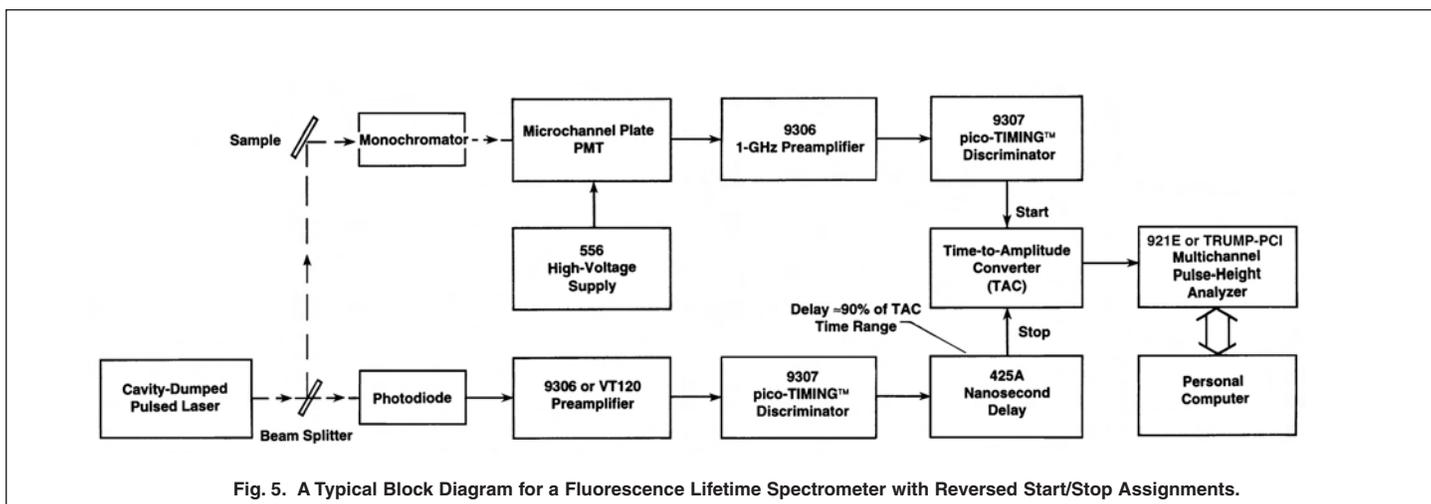


Fig. 5. A Typical Block Diagram for a Fluorescence Lifetime Spectrometer with Reversed Start/Stop Assignments.

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Limiting the Counting Rate to Avoid Spectrum Distortion

A high-resolution TAC measures the time interval from the first accepted start pulse to the next stop pulse. It ignores all subsequent start pulses and any additional stop pulses until it has finished converting the first pair of start and stop pulses. If either input is receiving randomly distributed pulses at a very high counting rate, the TAC will prefer to analyze the pulses arriving earlier on that input and will suppress the pulses that arrive later. This will distort the measured time spectrum for correlated start and stop events. The distortion can be controlled by limiting the counting rates at the start and stop inputs. From Poisson statistics,¹ it can be shown that limiting the average random counting rate r at both start and stop inputs to

$$r < 0.01 / T_{\text{range}} \quad (3)$$

will ensure that the number of suppressed pulses in the analyzed time range T_{range} will be less than 0.5% of the number of accepted pulses on the respective input. This condition is adequate to ensure less than a 1% distortion of the time spectrum.

For a short time range, $T_{\text{range}} = 50$ ns, the condition in Equation 3 limits the counting rate to 200,000 counts/s at both the start and stop inputs to the TAC. This counting rate is still high enough to require an MCA with a conversion time of 5 μ s or less in order to keep up with the data from the TAC.

When an MCS is a Better Choice than a TAC

A time-to-amplitude converter is a productive solution for measurements on time ranges less than 10 μ s when time resolutions from 10 ps to 50 ns are required. However, a TAC can measure only a single time interval for each start pulse, and this limits its utility on the longer time ranges. For example, the condition in Equation 3 restricts the input rates to <1,000 counts/s on a 10- μ s time range. This is a low data acquisition rate. On a 1-ms time range, the input rate is limited to 10 counts/s, an extremely low data acquisition rate! Obviously, a time-to-amplitude converter is handicapped by low data acquisition rates on the longer time ranges when distortion of the time spectrum must be avoided.

Most measurements that require time ranges in excess of 10 μ s involve a controlled, pulsed source of excitation. In such circumstances, a multichannel scaler (MCS) is advantageous because it can accept multiple stop pulses for each start pulse. The pulsed excitation source starts the time scan on the MCS, and the events caused by the excitation are counted as a function of time on the counting input of the MCS. The result is a spectrum of the number of events versus the time after excitation. With a pulse-pair resolving time of 1 ns, the ORTEC Model 9353 is able to process average "stop" rates up to 10 MHz with less than 1% dead time losses, and burst rates up to 1 GHz. Of course, the period between excitation (start) pulses must be longer than the time interval being measured.

Clearly, the MCS is the more productive instrument for measuring time ranges longer than a few microseconds. However, the performance for some MCS models on shorter time ranges is limited by the intrinsic time resolution off set by the minimum possible dwell time.

A TAC combined with the CAMAC multi-parameter ADCs is an ideal solution for measurements requiring correlated sampling of amplitude and time data from one or more detectors. The 9353 is not suited for multi-parameter measurements.

Generally, one should consider a TAC for time ranges <1 μ s and multi-parameter measurements and the 9353 for time ranges from microseconds to milliseconds. For further information on the latter two instruments see the Counters, Ratemeters, and Multichannel Scalers introduction.

Calibrating the Time Scale

The simplest way to calibrate the time scale of the spectrum recorded on the multichannel analyzer is to insert cable delays of known length between the timing discriminator output and the TAC input. The additional delay will shift the peak in the time spectrum. The amount of shift can be calibrated against the known value for the inserted delay. The Model 425A Nanosecond Delay is a convenient source of adjustable delays for this purpose.

For higher accuracy in calibrating the time scale, the Model 462 Time Calibrator is the better choice. This unit uses an accurate digital clock to produce stop pulses at precisely spaced intervals after a start pulse. A short data acquisition with the Model 462 connected to the TAC inputs results in multiple peaks in the spectrum. The spacing between these peaks corresponds to the period selected by the controls on the Model 462.

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Accounting and Correcting for Dead Time in the TAC and MCA

The sources of dead time in a time spectrometer employing a TAC and MCA are easily identifiable, although the derivation of the throughput equations is somewhat more complicated. The time-to-amplitude converter is only able to process one pair of start and stop pulses in each conversion. Once a start pulse has been accepted all further start pulses are ignored until the conversion and reset processes are finished. Similarly, the TAC responds to the first stop pulse that arrives after the accepted start pulse, and ignores all subsequent stop pulses until the next valid start pulse has been accepted. As a result, subsequent start pulses find the start input to be dead from the time of acceptance of the last valid start pulse until the end of the TAC reset. Additional stop pulses find the stop input to be dead from the time the first stop pulse is accepted (following a valid start pulse) until the time of acceptance of the next start pulse.

If the multichannel analyzer dead time is longer than the TAC dead time, the MCA can also contribute to the dead time losses, because the MCA will not always be ready to accept the next TAC output. Choosing an MCA conversion time that is less than the minimum TAC dead time eliminates the MCA dead time contribution. If the MCA dead time is longer than the TAC dead time, one can gate off the TAC start input with the MCA busy signal in order to use the throughput equations developed below.

The following throughput equations relate the time spectrum viewed by the detector to the spectrum actually recorded by the TAC and MCA. They can be used for three purposes: a) to predict the distortions caused by dead time losses, b) to determine the counting rate limits that render the distortion negligible or, c) to implement dead time correction algorithms that permit data acquisition at higher counting rates. The four most common cases are summarized below.

Case 1: Periodic Start and Random Stops, $T_s > T_d$

To avoid excessive complication, consider a periodic start pulse whose period T_s is longer than the combined TAC/MCA dead time T_d . In this case, no start pulses occur when the TAC/MCA cannot respond. The start pulse normally corresponds to the time at which a process is stimulated. The stop input is used to record the time spectrum of the products emitted from that stimulation. The apparatus must be designed to restrict the intensity of the product events so that statistical sampling of the time distribution is possible via single-ion or single-photon counting.

The MCA sorts the analog output of the TAC into a histogram, whose length is equal to the maximum number of channels offered by the MCA. Thus, each channel spans a time interval, Δt , and the start-to-stop time represented by channel i is

$$t = i \Delta t \quad (4)$$

where i extends from $i = 0$ to $i = i_{\max}$. The maximum channel number i_{\max} is typically in the range of 1000 to 16,000.

To demonstrate the minor effect of the detector and timing discriminator dead time, a single, extending dead time, T_e , will be ascribed to that source. T_e is represented in channel numbers by τ_e (rounded to the nearest integer value), where

$$T_e = \tau_e \Delta t \quad (5)$$

If a time spectrum is accumulated for a preset number of valid start pulses, n_1 , and the number of events recorded in channel i is q_i , then the probability of recording an event in channel i for a single valid start pulse is given² by equation (6).

$$\frac{q_i}{n_1} = \frac{Q_i}{n_1} \exp \left[-\sum_{j=0}^{i-1} Q_j / n_1 \right] \exp \left[-U(\tau_e - i) \sum_{j=i-\tau_e}^{-1} Q_j / n_1 \right] \quad (6)$$

The right-hand side of equation (6) is composed of three probabilities. The probability of an event impinging on the detector and destined for channel i (before dead time losses) is Q_i / n_1 . This event cannot be recorded in channel i if it was preceded by any stop events since the start pulse. The probability of no stop pulses from channel $j = 0$ to $i - 1$ is given by the first exponential term in equation (6). If the counting rate at the stop input is absolutely zero for $i < 0$ (no stop pulses preceding the start pulse) the last exponential term in equation (6) becomes 1. However, most detectors have some low level of background counting rate caused by thermal excitation. Hence, a background stop pulse occurring in the interval from $t = -T_e$ to $t = 0$ would prevent the desired stop pulses from being detected in the interval from $t = 0$ to $t = T_e$. To account for this effect, the last exponential term in equation (6) is the probability of no stop pulses preceding $i = 0$ in the time interval τ_e . The step function is defined by

$$U(\tau_e - i) = \begin{cases} 1 & \text{for } \tau_e - i > 0 \\ 0 & \text{for } \tau_e - i \leq 0 \end{cases} \quad (7)$$

²D.A. Gedcke, Development notes and private communication, Nov.–Dec. 1996.

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Equation (6) can be used to correct the acquired spectrum, q_i , for dead time losses in order to generate the corrected time spectrum, Q_i . One starts at channel 0 and presumes all Q_j preceding channel 0 are zero. As one moves channel by channel to the right in the spectrum the Q_j become available from the Q_i calculated for the previous channels. This calculation is repeated until the maximum channel, i_{\max} , has been treated. The resulting set of Q_i is the time spectrum corrected for dead time losses, with one exception. Because the values of Q_i for $i \leq 0$ were unknown and presumed zero, the corrected spectrum will be underestimated for values of i up to several times τ_e . This shortcoming can be easily overcome by adding sufficient cable delay to the stop input to move the spectral features of interest out of the affected region. This allows one to ignore the timing discriminator dead time if it is small compared to the measured time span.

Because the counts q_i are sampled from a preset number of start pulses, n_1 , the statistical variance in q_i is given by²

$$\sigma_{q_i}^2 = n_1 \frac{q_i}{n_1} \left(1 - \frac{q_i}{n_1}\right) \quad (8)$$

$$\approx q_i \quad \text{for } q_i / n_1 \ll 1$$

Moreover, the variance in the sum of the counts from any channels from $j = h$ to k is

$$\sigma_m^2 = m = \sum_{j=h}^k q_j \quad (9)$$

By using a straight-forward propagation-of-errors computation, while ignoring the timing discriminator dead time, the variance in the Q_i calculated via equation (6) is²

$$\sigma_{Q_i}^2 = Q_i (Q_i / q_i) \left[1 + (q_i / n_1) \sum_{j=0}^{i-1} \sigma_{Q_j}^2 / n_1\right] \quad (10)$$

$$\approx Q_i (Q_i / q_i)$$

The approximation in the last line of equation (10) is highly accurate, because the second term in the square brackets is negligible compared to 1 for practical applications. An alternative expression of the relationship in equation (10) is

$$\frac{\sigma_{Q_i}}{Q_i} = \frac{\sigma_{q_i}}{q_i} = \frac{1}{q_i^{1/2}} \quad (11)$$

In other words, the relative standard deviation in the calculated counts Q_i is determined by the relative standard deviation in the measured counts q_i .

Case 2: Random Start and Periodic Stop, $T_s > T_d$

Case 2 arises from the same application as Case 1, except the Reversed Start/Stop method is employed to reduce the TAC/MCA dead time. As described earlier with reference to Figure 5, the periodic stimulation pulse is delayed by a time interval D and applied to the TAC stop input. The delay is typically 90 to 95% of the time span selected on the TAC. The detected pulses from the products of the stimulation are fed to the start input.

The delay D is expressed in terms of a number of channels by

$$D = \delta \Delta t \quad (12)$$

where δ is rounded to the nearest integer value.

If there truly are no detected product events before the time of the original stimulation pulse, then the probability of recording an event in channel i for a single stimulation pulse is

$$\frac{q_i}{n_2} = \frac{Q_i}{n_2} \exp\left(-\sum_{j=i+1}^{i_{\max}} Q_j / n_2\right) \quad (13)$$

where q_i is the number of events recorded in channel i as a result of n_2 stimulation pulses. Note that n_2 is the number of delayed stimulation pulses presented to the stop input, whether or not they were accepted by the stop input. It is presumed that the period

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between stimulation pulses, T_s , is longer than the TAC/MCA dead time, T_d , so that the TAC and MCA are always ready to process the events from the next stimulation pulse. (See Case 3 for the opposite situation: $T_s < T_d$.)

The probability of a recorded event is composed of two probabilities on the right-hand side of equation (13). The probability of an event arriving at the detector at a time destined to be categorized in channel i is Q_i / n_2 . The exponential term describes the probability that no start pulses will precede the desired start pulse in the time interval between the undelayed stimulation pulse and the arrival time of the start pulse in channel i . Because of the reversal of the start and stop inputs, the summation in the exponential must extend from $j = i + 1$ to $j = \delta$. For convenience, the summation has been extended past $j = \delta$ to $j = i_{max}$. If there truly were no detected start events prior to the undelayed stimulation pulse, the counts will be zero for all channels from δ to i_{max} .

To calculate the corrected counts, Q_i , from the measured counts, q_i , equation (13) must be applied by starting at i_{max} and working channel by channel to $i = 0$. Thus, the values needed for Q_j are available from the Q_i already calculated for higher channel numbers.

If there are significant uncorrelated background pulses arriving at the start input prior to the undelayed stimulation pulse the modification to equation (13) can be rather complicated.² One can avoid this complication by holding the start input gate closed until the undelayed stimulation pulse occurs. The start input gate is opened only for the interval from the occurrence of the undelayed stimulation pulse until the arrival of the delayed stimulation pulse at the stop input. This permits the valid application of equation (13). In practice, a delay of the order of T_e may need to be inserted in the stop input to shift the prompt portion of the spectrum clear of the gating at $i = \delta$.

As for Case 1, the statistical variance in the recorded counts is

$$\sigma_{q_i}^2 = q_i \quad (14)$$

The variance in the calculated corrected counts is

$$\sigma_{Q_i}^2 = Q_i (Q_i / q_i) \quad (15)$$

and

$$\frac{\sigma_{Q_i}}{Q_i} = \frac{\sigma_{q_i}}{q_i} = \frac{1}{q_i^{1/2}} \quad (16)$$

Case 3: Random Start and Periodic Stop, $T_s < T_d$

This case is the same as Case 2, except that the period between stimulation pulses, T_s is less than the TAC/MCA dead time, T_d . Fluorescence lifetime spectrometry (Fig. 5) is a typical application. For simplicity in demonstrating the critical points, the discriminator dead time, T_e , is ignored. If q_i is the number of events recorded in channel i for n stimulation pulses, then the probability of recording an event in channel i for a single stimulation pulse is²

$$\frac{q_i}{n} = \frac{Q_i}{n} \exp\left(-\sum_{j=i+1}^{\tau_s} Q_j/n\right) [1 - \beta_I \sum_{k=0}^{\tau_s} q_k/n - U\{i - (1 - \beta_F) \tau_s\} \sum_{k=0}^{\tau_s} q_k/n] \quad (17)$$

The channel summation limit, τ_s , is defined by

$$T_s = \tau_s \Delta t \quad (18)$$

and τ_s is rounded to the nearest integer value.

The right-hand side of equation (17) consists of three probability factors. The first two are the same as in Case 2, except that n_2 has been replaced with n , and the summation limit is set by the period between stimulation pulses, τ_s . (It is presumed that the time span of the TAC is selected to be slightly longer than τ_s .) The third factor consists of the terms in the square brackets, and this factor represents the probability of not accepting start events because the TAC/MCA is busy processing a previous event.

The dead time of the TAC and MCA can be written as the sum of the variable, measured, start-to-stop time, t_{ss} , and the constant processing time, t_d . (A constant conversion-time MCA is presumed.)

$$T_d = t_{ss} + t_d \quad (19)$$

Note that t_d always begins on an accepted stop pulse and ends when the TAC/MCA combination can accept the next start pulse. (It is presumed that the MCA Busy signal gates the TAC Start Input.)

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It is convenient to express the results in terms of β , which is the ratio of t_d to T_s .

$$t_d = \beta T_s = (\beta_I + \beta_F) T_s \quad (20)$$

where β_I is the integer part of β , and β_F is the fractional part of β . With this definition in mind, the terms in the square brackets in equation (17) are explained as follows.

The second term in the square brackets is the probability that an event has been accepted in the previous β_I intervals of T_s , causing the TAC/MCA to be busy when the desired start pulse arrives. The third term is the same probability, but for interval number $\beta_I + 1$ prior to the desired start pulse. This latter interval is important because it generates a busy period, t_d , that extends by an amount $\beta_F T_s$ into the period that contains the desired start pulse. Consequently, only the earlier start pulses in the desired start-pulse interval are suppressed by this term. That fact is described in equation (17) by the unit step function

$$U\{i - (1 - \beta_F) \tau_s\} = 1 \text{ for } i > (1 - \beta_F) \tau_s \\ = 0 \text{ for } i \leq (1 - \beta_F) \tau_s \quad (21)$$

This third term in the square brackets causes a distortion of the spectrum that is extremely difficult to correct, because it is difficult to measure and predict $\beta_F \tau_s$. The practical solution is to restrict the counting rate so that the error caused by the third term is less than 1%. This restriction requires

$$\sum_{k=0}^{\tau_s} q_k / n < 0.01 \quad (22)$$

Note that equation (22) also guarantees that the distortion expressed by the exponential term in equation (17) will be $<1\%$. It also ensures that the dead time effects of the timing discriminator are negligible, provided $T_e < T_s$.

For efficient throughput¹ the TAC/MCA dead time losses should be restricted to $<50\%$. Because the second term in the square brackets dominates the dead time losses, this leads to the second restriction

$$\beta_I \sum_{k=0}^{\tau_s} q_k / n < 0.5 \quad (23)$$

which typically requires $\beta_I < 50$. The restrictions in equations (22) and (23) are easy to check by summing the counts recorded in the time spectrum and dividing by the corresponding number of stimulation pulses.

Clearly, Case 3 does not lead to a practical correction algorithm. Instead, equations (22) and (23) define the limits on the operating parameters necessary to avoid distortion. If it is sufficient to simply measure the shape of the time spectrum one can verify that conditions (22) and (23) are met and then use the recorded spectrum, q_i .

If the absolute value of Q_i / n is required, one can apply a simple live time clock that turns off whenever the TAC/MCA combination is unable to respond to a start pulse. This will require feeding the TAC Busy signal to the MCA live time clock and connecting the MCA Busy signal to the Start Input Gate on the TAC so that the TAC/MCA combination is dead whenever the TAC or the MCA is busy. The live time clock corrects for the dominant dead time losses caused by the second term in the square brackets in equation (17). Under conditions (22) and (23) all other losses and distortion will be $<1\%$. The basic principle of the live time clock¹ is expressed by

$$\frac{Q_i}{t} = \frac{q_i}{t_L} \quad (24)$$

Dividing the counts, q_i , recorded in the live time, t_L , yields the corrected event rate, Q_i / t . It follows that

$$Q_i / n = (Q_i / t) / (n / t) = (q_i / t_L) / (n / t) \quad (25)$$

In other words, one divides the recorded counts by the livetime and by the known repetition rate of the stimulation pulses, n / t , in order to calculate Q_i / n . Because the q_i events are counted for a preset live time, the relative standard deviation in q_i , Q_i , and Q_i / n is given by equation (16)¹.

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Case 4: Random Starts and Random Stops

Random events are typically encountered at both the start and stop inputs when it is not possible to periodically stimulate the process to be measured. An example is the measurement of the lifetime of an excited state in a nucleus when the excited state is populated as the result of radioactive decay. For example, consider the emission of an alpha particle from a radioactive sample signaling the decay which forms the excited state, followed by the emission of a gamma ray marking the decay of the excited state to the ground state. The alpha particle detector supplies the pulse for the TAC start input, and the gamma ray detector feeds the stop input. Since the detection probability for both types of radiation is modest, there is a moderate probability that 1) a start event will be detected without detecting the correlated stop pulse, 2) a stop pulse will be detected without detecting the correlated start event, and 3) an uncorrelated pair of start and stop events will be recorded. These actions can cause dead time or uncorrelated background in the measured time spectrum.

If it is sufficient to measure the correct shape of the decay curve to extract the lifetime, then equations (4) through (11) of Case 1 provide an adequate description of the measurement. If the absolute probability of detecting a particular start-to-stop time interval is also required, the effect of dead time losses for the start input must be accounted for.

If the start events are randomly and uniformly distributed in time (constant counting rate), the throughput relationship is expressed by^{1,2}

$$\frac{N_1}{n_1} = \exp(R_1 T_e) + U(T_d - T_e) R_1 (T_d - T_e) \quad (26)$$

where N_1 is the number of start events at the detector (before dead time losses) and n_1 is the number of start pulses accepted by the TAC/MCA combination. $U(T_d - T_e)$ is the previously defined step function, and R_1 is the counting rate of start events at the detector, i.e.,

$$R_1 = \frac{N_1}{t} \quad (27)$$

Normally $T_e \ll T_d$, and equation (26) simplifies to the form for non-extending dead time.

$$\frac{N_1}{n_1} = 1 + R_1 T_d = \frac{1}{1 - r_1 T_d} \quad (28)$$

where

$$r_1 = \frac{n_1}{t} \quad (29)$$

The simplest way to account for the relation in equation (28) is to use a simple livetime clock that turns off for the combined dead time of the TAC and MCA. The relationship between live time, t_L , and real time, t , is given by¹

$$\frac{n_1}{t_L} = \frac{N_1}{t} = R_1 \quad (30)$$

Consequently, the joint probability of detecting a start pulse and a stop pulse such that the start-to-stop time interval is destined for channel i is

$$P_i = R_1 \frac{Q_i}{n_1 \Delta t} = \frac{n_1}{t_L} \frac{Q_i}{n_1 \Delta t} = \frac{Q_i}{t_L \Delta t} \quad (31)$$

The division by t_L and Δt expresses both the start and stop probabilities on a per-unit-time basis.

If the live time, t_L , required to record n_1 accepted start pulses is measured, the relative standard deviation in t_L is given by¹

$$\frac{\sigma_{tL}}{t_L} = \frac{\sigma_{n1}}{n_1} = \frac{1}{(n_1)^{1/2}} \quad (32)$$

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Applying a propagation-of-errors calculation leads to the expression for the relative standard deviation in P_i

$$\frac{\sigma_{P_i}}{P_i} = \left[\frac{1}{n_1} + \frac{1}{q_i} \right]^{1/2} \quad (33)$$

Because $q_i \ll n_1$, the relative standard deviation in equation (33) will be dominated by q_i .

Specifications subject to change
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