

Choosing the Right Counting Solution

ORTEC offers a variety of instruments to configure counting systems for many applications, from simple to complex. The following descriptions and selection charts will help identify the functions and data interfaces needed for the application.

The Basic Functions of Counting Systems

There are five general classes of counting systems: counters, timers, ratemeters, multichannel scalars and time digitizers, and digital signal averagers. ORTEC offers four of the five. Counters simply count the number of input pulses received during the counting period. Timers count pulses generated by an internal clock and are used to measure elapsed time or to establish the length of the counting period. Ratemeters provide a meter readout of the pulse count rate and convert this frequency to a DC voltage or current proportional to the average count rate per unit of time which is normally expressed in counts per second (counts/s).

All ORTEC NIM timers contain preset controls to establish the duration of the counting period. When counting is initiated, the internal clock pulses are counted until the preset condition is reached; at that time, counting is stopped in all counters connected to the common gate line of the master timer. If the external input is used, the preset control will apply to counting of the pulses at the external input and will result in preset count operation.

Multichannel scalars count the number of events that occur during the time interval t to $t + \Delta t$ as a function of time. The interval Δt is called the dwell time. The time t is quantized into channels or bins by the relation $t = i \Delta t$, where i is the bin number (an integer). Dwell times can be selected from fractions of a nanosecond to hours, and the total number of bins ranges from 4 to 67 million. When the scan is started, the MCS begins counting input events in the first channel of its digital memory. At the end of the selected dwell time the MCS advances to the next channel of memory to count the events. This dwell and advance process is repeated until the MCS has scanned through the preset number of memory channels. The display shows a graph of counts versus dwell channels from zero time on the left to end of the scan on the right.

Both multichannel scalars and time digitizers are triggered by a start pulse that defines zero time. Typically, the start pulse corresponds to the stimulation of a process. The "products" of the process are counted as a function of time by the MCS, or their emission times are measured by the time digitizer. In a single pass through the selected time span, both instruments can record multiple events. If the process is repeatable, the data from multiple passes through the time span can be summed to improve the statistical precision. The final result is a histogram showing the probability of observing the product events as a function of time.

MCS vs. a Counter and Timer

Although a computer-controlled counter and timer like the 928 can be used to read counting rate as a function of time, the MCS technology offers several important advantages. The time taken to read, clear, and start a conventional counter at the end of each counting interval can range from microseconds to milliseconds. This dead time causes significant gaps in the recorded data. High performance multichannel scalars have negligible dead time between counting intervals (channels), and this avoids blank regions in the recorded time profile. Conventional counters and timers rarely handle time intervals shorter than 10 ms, whereas multichannel scalars are available with minimum dwell times ranging from 2 μ s down to 5 ns. Furthermore, most MCS products include standard operating software to acquire, display, and manipulate the data. With a conventional counter and timer one must develop custom software for the intended application.

Multiple Stop Advantages over a Time-to-Amplitude Converter (TAC)

A MCS can operate as a multiple-stop time spectrometer. On each scan through the selected time span, the MCS can record multiple stop events. This is achieved by counting "stop" events in the appropriate bin as they arrive. In contrast, a time-to-amplitude converter (TAC) can record only one stop event for each start trigger.

Multichannel scalars have an upper limit on the event rate as a result of the dead time caused by the pulse-pair resolving time. Generally, the probability of detecting a single event within the pulse-pair resolving time must be kept less than 2% during each scan, in order to limit the dead time losses and distortions to less than 1%. This places an unproductive limit on the ion rates that can be accommodated in the application of Time-of-Flight Mass Spectrometry (TOF-MS) when analyzing the output of a chromatograph (LC or GC). As a result, TOF-MS spectra collected for 100-ms time intervals contain circa 20 counts in the largest peaks. This leads to statistical errors in excess of 22%, and correspondingly high detection limits. A Digital Signal Averager would be a much more appropriate solution for this type of application.

Accounting and Correcting for Dead Time Effects

With both multichannel scalars and time digitizers, dead times in the counting system can cause distortion of the measured time spectrum. One means of minimizing the distortion is to operate at counting rates low enough to keep dead time losses below 1%. Usually this strategy ensures that the spectrum distortion will be <1%. To achieve this goal one must know the equations linking counting rate, dead times, and dead time losses. Fortunately, these equations lead one to correction algorithms that can be applied in several practical cases to enable operation at significantly higher counting rates.

Counters/Ratemeters/ Multichannel Scalers

Case 1: $\Delta t \ll T_d$

Cascaded Dead Times:

The dead time experienced in the counting chain is typically composed of two cascaded components, T_e and T_{ne} . T_e is the extending dead time caused by the duration of the analog signal from the detector at the noise threshold of the timing (or counting) discriminator. It is an extending dead time because a second analog pulse occurring during a preceding pulse extends the dead time by one pulse width from the arrival time of the second pulse. The non-extending dead time, T_{ne} , can be caused by the pulse width of the discriminator output driver, or it can be a longer dead time contributed by a circuit in the MCS or time digitizer. Sufficient accuracy^{1,2} will be achieved if one chooses the longer of these two dead times to represent T_{ne} . A second pulse occurring during T_{ne} is ignored and does not affect the dead time. It is convenient to define the approximate dead time in the system as

$$T_d \approx T_e + U(T_{ne}-T_e) (T_{ne}-T_e) \quad (1)$$

where $U(T_{ne}-T_e)$ is a unit step function defined by

$$\begin{aligned} U(T_{ne}-T_e) &= 1 \text{ for } T_{ne} > T_e \\ &= 0 \text{ for } T_{ne} \leq T_e \end{aligned} \quad (2)$$

Under that definition, the equations for Case 1 are valid if the quantization interval, Δt , is insignificant compared to T_d .

Presume a time digitizer that has summed the repetitive spectra from n start triggers. The counts in the i th bin of the resulting spectrum (after suffering dead time losses) are defined to be q_i , and the time, t , is related to the bin number by

$$t = i \Delta t \quad (3)$$

By analogy to equation (3) it is convenient to define the quantized dead times, τ_e , τ_{ne} , and τ_d , by equations (4).

$$T_e = \tau_e \Delta t \quad (4A)$$

$$T_{ne} = \tau_{ne} \Delta t \quad (4B)$$

$$T_d = \tau_d \Delta t \quad (4C)$$

Note that i , τ_e , τ_{ne} and τ_d are all rounded to integer values.

The number of counts that would have been recorded in bin i if the dead time were zero is defined to be Q_i . The distorted spectrum recorded in the measurement is represented by q_i , whereas Q_i is the undistorted spectrum that is sought.

When the counting rates are low enough to yield single-ion or single-photon counting, one can apply statistical sampling theory. Poisson Statistics can also be applied directly, provided the dead time losses are negligible³.

In equation (5), q_i/n is the probability of recording an event in the i th bin during a single pass through the time span. It is composed of three probabilities⁴, as described in the right hand side of the equation.

Cascaded Dead Time Equation:

$$\frac{q_i}{n} = \frac{Q_i}{n} \exp \left\{ - \sum_{j=i-\tau_e}^{i-1} \frac{Q_j}{n} \right\} \left[1 - U(\tau_{ne} - \tau_e) \sum_{j=i-\tau_{ne}}^{i-\tau_e-1} \frac{q_j}{n} \right] \quad (5)$$

The first term, Q_i/n is the probability that an event will arrive at the detector at a time suitable to be categorized in bin i . In order to be recognized as distinct from previous analog pulses there must be no pulses arriving at the detector in the time interval τ_e preceding the pulse for bin i . That probability is given by the exponential term in equation (5). If a pulse had been counted by triggering the non-extending dead time in the time interval τ_{ne} preceding the pulse for bin i , the pulse for bin i would be lost. Consequently, the term in the square brackets is the probability that no pulses are recorded in the preceding time interval τ_{ne} . Note that the sum stops at $j = i - \tau_e - 1$ because the exponential term already guarantees no pulses occurred in the time interval from $j = i - \tau_e$ to $i - 1$.

Equation (5) can be rearranged to get the formula for computing the corrected spectrum, Q_i , from the distorted spectrum, q_i .

$$Q_i = \frac{q_i}{\exp \left\{ - \sum_{j=i-\tau_e}^{i-1} \frac{Q_j}{n} \right\} \left[1 - U(\tau_{ne} - \tau_e) \sum_{j=i-\tau_{ne}}^{i-\tau_e-1} \frac{q_j}{n} \right]} \quad (6)$$

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One applies the correction algorithm by starting at bin $i = 0$, while presuming that q_i , q_j , and Q_j are all zero for negative values of i and j . For each i , the value Q_i is calculated from equation (6) using the recorded values q_i and q_j along with the Q_j values calculated for the previous values of i . This correction calculation is applied bin by bin until the maximum bin in the spectrum has been treated. At that point the list of Q_i values is the corrected spectrum.

If there truly were no counts to be detected for negative values of i , then the Q_i data near $i = 0$ will represent the true spectrum before dead time losses. If the detector was actually responding to events for negative values of i , then Q_i will be underestimated until the bin number exceeds several times τ_d . Frequently, this shortcoming can be eliminated by inserting a coaxial cable delay of the appropriate length between the detector and the stop input on the time digitizer.

Single, Extending Dead Time:

For a system where the detector pulses are longer than any other dead times in the system ($\tau_e > \tau_{ne}$), equation (5) simplifies to the equation for a single, extending dead time⁴.

$$q_i = Q_i \exp \left\{ -\sum_{j=i-\tau_e}^{i-1} Q_j / n \right\} \quad (7)$$

Single, Non-extending Dead Time:

The other extreme is a system in which detector pulse widths are negligible compared to the non-extending dead time in the MCS or time digitizer. In that case, equation (6) simplifies to^{4,5}

$$Q_i = \frac{q_i}{1 - \sum_{j=i-\tau_{ne}}^{i-1} q_j / n} \quad (8)$$

Accuracy of the Dead Time Correction:

It can be demonstrated by substituting known values into equations (6), (7) and (8) that all three equations yield predictions of Q_i / q_i that are within 1% of each other provided $Q_i / q_i < 1.15$ (i.e., a dead time correction $< 15\%$), and provided τ_d is substituted for the single dead times in equations (7) and (8). This allows a simpler correction algorithm to be implemented using equation (8). In fact, the algorithm using equation (8) can start at the maximum value of i and proceed towards $i = 0$, while replacing q_i with Q_i in the data file.

Without the dead time correction algorithm, one would have to limit the counting rate to achieve $< 1\%$ dead time losses in order to limit the spectrum distortion to $< 1\%$. By applying the dead time correction algorithm, one can typically operate at a factor of 10 higher dead time loss, while still achieving $< 1\%$ spectrum distortion. This implies a factor of 10 higher data rates. However, the accuracy of the correction is limited by the factors discussed next.

If the time spectrum is constant across all bins, it is easy to show that a 10% error in the assumed value for the dead time in equations (6), (7), or (8) will lead to $< 1\%$ error in the corrected counts if $Q_i / q_i < 1.10$. A more serious case is a narrow peak centered at bin i , and preceded by an intense, narrow peak centered at bin $j = i - \tau_d$. A small error in the presumed value for τ_d can result in either including or excluding the peak at bin j in the dead time corrections. This can make a large difference in the dead time correction applied to the peak at bin i . An additional issue is the error in rounding off the presumed dead time to the nearest integer value. This leads to round-off errors at the two limits of the sums in the equations. That effect can be restricted to an error $< 1\%$ if one ensures that $q_j / n < 0.005$ for all j . Clearly, it is important to use an accurately measured dead time in the correction formula.

By applying basic probability theory, it can be shown that the statistical variance in the recorded counts q_i is given by⁴

$$\begin{aligned} \sigma_{q_i}^2 &= q_i (1 - q_i / n) \\ &\approx q_i \quad \text{for } q_i / n \ll 1 \end{aligned} \quad (9)$$

Moreover, for the sum of the recorded counts in any number of bins, such as

$$M = \sum_{j=i-\tau}^{i-1} q_j \quad (10)$$

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the statistical variance in the sum is

$$\sigma_M^2 = M = \sum_{j=i-\tau}^{i-1} q_j \quad (11)$$

A straight forward propagation-of-errors computation predicts the statistical variance in the corrected counts Q_i calculated from equation (8) to be^{4,5}

$$\sigma_{Q_i}^2 = Q_i k_i \quad (12)$$

where k_i defines the magnification factor arising from the variances in the q_i and q_j in equation (8), i.e.,

$$k_i = (Q_i / q_i) + (Q_i / q_i)^2 [(Q_i / q_i) - 1] (q_i / n) \quad (13)$$

The effect of k_i is small for dead time corrections $Q_i / q_i < 1.15$, but the second term of k_i escalates rapidly for higher dead time corrections.⁵

Case 2: $T_d \ll \Delta t$, with Counts Essentially Constant Across Δt

This solution is applicable to multichannel scalars. Typically the dead time T_d is < 20 ns. If the dwell time is large compared to 20 ns (for example: 2000 ns) and the counting rate varies insignificantly during the dwell time of a bin, then one can transform equation (6) into the form that applies for approximately constant counting rate.⁴ The result is

Cascaded Dead Times:

$$Q_i = \frac{q_i}{\exp\left\{\frac{-Q_i T_e}{n \Delta t}\right\} \left[1 - U(T_{ne} - T_e) \frac{q_i (T_{ne} - T_e)}{n \Delta t}\right]} \quad (14)$$

Note that Q_i appears on both sides of the equation. So equation (14) must be solved by iteration. If one substitutes the instantaneous counting rates defined by equation (15) in equation (14), the constant counting rate formula are generated.

$$R_i = \frac{Q_i}{n \Delta t} \quad (15a)$$

$$r_i = \frac{q_i}{n \Delta t} \quad (15a)$$

In some applications the detector pulse width will exceed the non-extending dead time ($T_e > T_{ne}$) leading to

Single, Extending Dead Time:

$$q_i = Q_i \exp\left\{\frac{-Q_i T_e}{n \Delta t}\right\} \quad (16)$$

If the detector pulse width is negligible compared to the non-extending dead time, equation (14) becomes

Single, Non-extending Dead Time:

$$Q_i = \frac{q_i}{1 - (q_i / n)(T_{ne} / \Delta t)} \quad (17a)$$

or

$$q_i = \frac{Q_i}{1 + (Q_i / n)(T_{ne} / \Delta t)} \quad (17b)$$

As in Case 1, it is convenient and adequate to use equation (17a) when the dead time losses are less than 15%, provided T_{ne} is replaced with T_d . Most of the caveats in Case 1 concerning accuracy apply here as well. The one exception is the statistical variance in the recorded counts q_i which is given by³

$$\sigma_{q_i}^2 = q_i (q_i / Q_i)^2 \quad (18)$$

for the case of non-extending dead time. The statistical variance in the Q_i calculated from equation (17a) is given by³

$$\sigma_{Q_i}^2 = Q_i (Q_i / q_i) \quad (19)$$

Note that large dead time corrections magnify σ_{Q_i} (the standard deviation in the corrected counts) by the square root of the correction factor, Q_i / q_i . Equations (18) and (19) are valid if $\Delta t \gg T_d$ and Δt is large compared to the mean spacing between pulses. The expressions for the σ_{q_i} and σ_{Q_i} corresponding to extending dead time are approximately the same as equations (18) and (19) for $Q_i / q_i < 1.1$, but diverge wildly from the non-extending case for large correction factors³. This provides an incentive for limiting dead time losses to $< 10\%$.

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Case 3: $T_d \sim \Delta t$

In this case there is not a practical algorithm for making accurate dead time corrections in the time histogram. However, one can use the equations above and the equations in the Fast Timing Discriminators section to predict the severity of the dead time losses and to determine the limitations on the counting rates that guarantee negligible dead time losses.

Case 4: Counting Rates Vary Significantly Across Δt

The statements under Case 3 apply.

¹ Jörg W. Müller, *Nucl. Instr. Meth.* 112, (1973), 47–57, Fig. 3.

² D.R. Beaman, et al., *J. of Physics E: Sci. Instr.* (1972), 5, 767–776.

³ Ron Jenkins, R.W. Gould, and Dale Gedcke, *Quantitative X-Ray Spectrometry*, Marcel Dekker, New York, (first edition), 1981, Chap. 4.

⁴ D.A. Gedcke, Development notes and private communication, Nov.–Dec. 1996.

⁵ P.B. Coates, *Rev. Sci. Instrum.* 63 (3), March 1992, 2084.

Counters/Ratemeters/ Multichannel Scalers

Ratemeters Selection Guide									
Model	Type	Full Scale Ranges		Input Types	Pulse-Pair Resolution	Audible Output	Recorder Output	Package and Width	Special Features
		Minimum	Maximum						
449	Log/Linear	10 counts/s	10 ⁶ counts/s	±Logic pulses >3 V high & >50 ns wide	<100 ns, or <1% of average pulse spacing	449-2 option	Yes	NIM-2	0 to 100% zero suppression
928	Log/Linear	1000 counts/s	10 ⁸ counts/s	Discriminator ± >3.5 ns wide	<70 ns	Yes	No	NIM-1	PC Control and Display. Programmable controls and output.
9349	Log/Linear	10 counts/s	10 ⁶ counts/s	Negative NIM >4 ns wide	<100 ns, or <1% of average pulse spacing	No	Yes	NIM-2	0 to 100% zero suppression

MCS Selection Guide	
Feature	EASY-MCS
Package	All-in-One USB connectable Box
Dwell Time	
Minimum	100 ns
Maximum	1300 s
Memory Length (channels)	65,536
Time Span (Full Memory Length)	
Minimum	6.55 ms
Maximum	2.7 years
Inputs and Rates	
Fast ± Discriminator	150 MHz
SCA	1 MHz
SCA or Discriminator Controls	Computer
SCA Sweep Mode	Yes
Dead Time Between Channels	Zero
End-of-Pass Dead Time	Zero
Ramp	Standard. Sawtooth or Triangular with Computer Adjustable Start/Mid/End Points
Software Base	Microsoft Windows
User-Defined Calibration of Horizontal Axis	Linear, Quadratic, or Cubic
Programmer's Toolkit	A11

Software Programmer's Toolkits

Each multichannel scaler and time digitizer includes a complete software package to operate the instrument in virtually all applications. Occasionally, it is advantageous to integrate the instrument into a software program specific to a unique application. ORTEC offers the Programmer's Toolkits summarized in the table below to facilitate special programming.

Toolkit	Supports	Operating Environment
A11 (CONNECTIONS Programmer's Toolkit)	EASY-MCS	Windows 2000/XP/7